1º a) $\int \frac{5}{7+10x^2} \cdot dx = \int \frac{5}{7\left(1+\frac{10x^2}{7}\right)} dx = \frac{5}{7}\int \frac{dx}{1+\frac{10x^2}{7}} =$

$= \frac{5}{7} \cdot \int \frac{1}{1+\left(\frac{\sqrt{10}\,x}{\sqrt{7}}\right)^2} dx = \frac{5}{7} \cdot \frac{\sqrt{7}}{\sqrt{10}} \cdot \int \frac{\frac{\sqrt{10}}{\sqrt{7}} \cdot dx}{1+\left(\frac{\sqrt{10}\cdot x}{\sqrt{7}}\right)^2} = \frac{5\sqrt{7}}{7\sqrt{10}} \arctan\left(\frac{\sqrt{10}\cdot x}{\sqrt{7}}\right)$

$= \frac{5\sqrt{70}}{70} \cdot \arctan\left(\frac{\sqrt{70}}{7} x\right) + C$

b) Por partes: $u = e^{2x}$, $dv = \operatorname{sen} x \cdot dx \Rightarrow \int e^{2x} \cdot \operatorname{sen} x \cdot dx =$

$= e^{2x} \cdot (-\cos x) - \int -\cos x \cdot 2 \cdot e^{2x} dx = -e^{2x} \cos x + 2 \int \cos x \cdot e^{2x} dx \Rightarrow$

$\{u = e^{2x},\ dv = \cos x \cdot dx\}$ $= -e^{2x} \cos x + 2\left[e^{2x} \cdot \operatorname{sen} x - \int \operatorname{sen} x \cdot 2 e^{2x} dx\right]$

$\Rightarrow I = \int e^{2x} \operatorname{sen} x \, dx = -e^{2x} \cos x + 2 e^{2x} \operatorname{sen} x - 4 I \Rightarrow$

$\Rightarrow 5 I = -e^{2x} \cos x + 2 e^{2x} \operatorname{sen} x \Rightarrow I = \int e^{2x} \operatorname{sen} x \, dx = e^{2x}\left(\frac{-\cos x + 2 \operatorname{sen} x}{5}\right) + C$

c) $\sqrt{x} = t \Rightarrow x = t^2 \Rightarrow dx = 2t \cdot dt \Rightarrow \int \frac{dx}{2x \cdot (x+\sqrt{x})} =$

$= \int \frac{2t\,dt}{2 \cdot t^2 (t^2+t)} = \int \frac{dt}{t\,(t^2+t)} = \int \frac{dt}{t^2(t+1)} = \int \left(\frac{A}{t^2} + \frac{B}{t} + \frac{C}{t+1}\right) dx \ (*)$

$\frac{1}{t^2(t+1)} = \frac{A}{t^2} + \frac{B}{t} + \frac{C}{t+1} = \frac{A(t+1) + B(t+1)\cdot t + Ct^2}{t^2(t+1)} \Rightarrow$

$\Rightarrow 1 = A(t+1) + B(t+1)\cdot t + Ct^2 \Rightarrow$ Para $t=0 \Rightarrow A=1$

Para $t=-1 \Rightarrow C=1$

Para $t=1 \Rightarrow 1 = 2+2B+1 \Rightarrow B=-1$

$\Rightarrow (*) = \int \frac{1}{t^2} dt + \int \frac{-1}{t} dt + \int \frac{1}{t+1} dt = \int t^{-2} dt - \int \frac{1}{t} dt + \int \frac{1}{t+1} dt =$

$= \frac{t^{-1}}{-1} - \ln|t| + \ln|t+1| = -\frac{1}{\sqrt{x}} - \ln|\sqrt{x}| + \ln|\sqrt{x}+1| + C$

d) $\int \frac{2}{senx \cdot cosx} dx = 2 \int \frac{1}{senx \cdot cosx} dx = 2 \int \frac{sen^2x + cos^2x}{senx \cdot cosx} dx$

$= 2 \int \left(\frac{sen^2x}{senx \cdot cosx} + \frac{cos^2x}{senx \cdot cosx} \right) dx = 2 \cdot \int \left(\frac{senx}{cosx} + \frac{cosx}{senx} \right) dx =$

$= 2 \cdot \left[\int tanx \, dx + \int cotan\, x \, dx \right] = 2 \cdot \left[-Ln|cosx| + Ln|senx| \right] =$

$= 2 \cdot Ln \left| \frac{senx}{cosx} \right| = \underline{2 \cdot ln|tanx| + C}$

e) $\int \frac{6x}{8 - 10x^2} dx = \int \frac{6x}{t} \cdot \frac{dt}{-20x} = -\frac{6}{20} \int \frac{dt}{t} =$

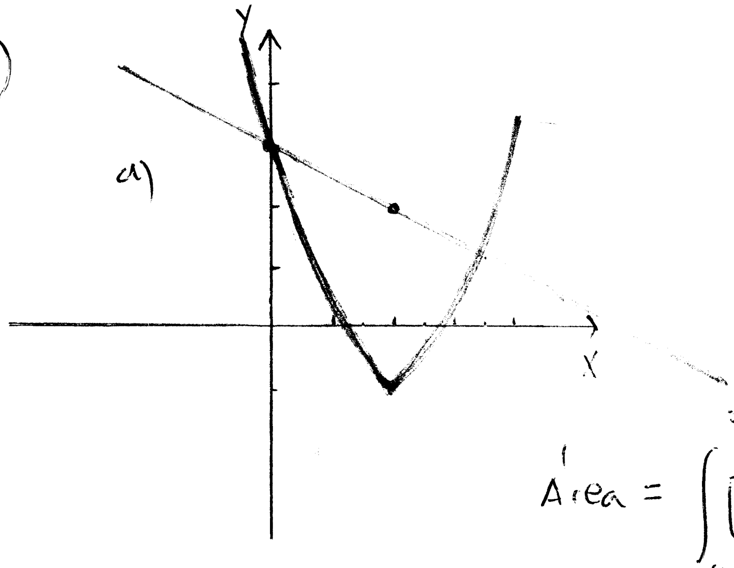
$t = 8 - 10x^2$

$dt = -20x \, dx$

$= -\frac{3}{10} Ln|t| = \underline{-\frac{3}{10} \cdot Ln|8 - 10x^2| + C}$

(20)

a)



b) $f(x) = y$

$x^2 - 4x + 3 = -\frac{x}{2} + 3$

$x^2 - 4x + \frac{x}{2} = 0$

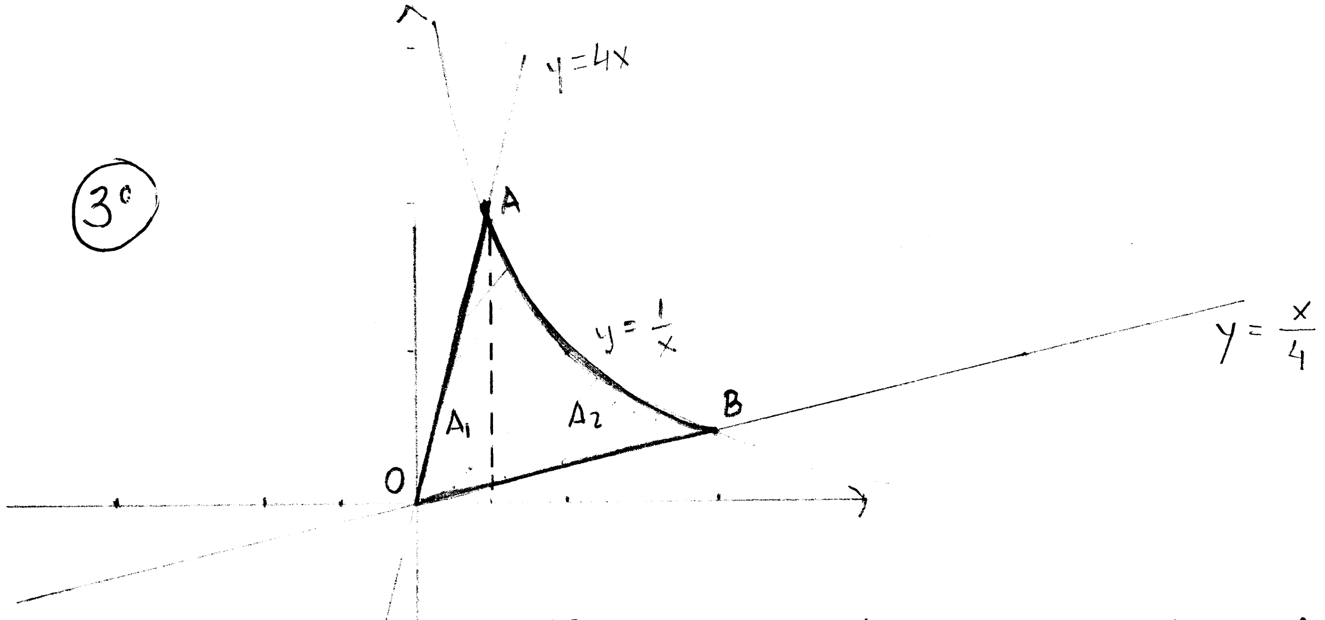
$2x^2 - 7x = 0 < \begin{matrix} x = 0 \\ x = \frac{7}{2} \end{matrix}$

$Area = \int_0^{7/2} \left[-\frac{x}{2} + 3 - (x^2 - 4x + 3) \right] dx =$

$= \int_0^{7/2} \left(x^2 + \frac{7x}{2} \right) dx = -\frac{x^3}{3} + \frac{7x^2}{4} \Big]_0^{7/2} = \left(-\frac{(7/2)^3}{3} + \frac{7 \cdot (7/2)^2}{4} \right) - (0) =$

$= -\frac{243}{24} + \frac{243}{16} = \frac{-486 + 729}{48} = \frac{343}{48} = \boxed{7'145 \, u^2}$

3°



Igualamos las tres funciones, dos a dos, para obtener los puntos de corte:

$$O:\ \begin{cases} y = 4x \\ y = \frac{x}{4} \end{cases} \Rightarrow O\ (0,0)$$

$$A:\ \begin{cases} y = 4x \\ y = \frac{1}{x} \end{cases} \Rightarrow 4x = \frac{1}{x} \Rightarrow 4x^2 = 1 \Rightarrow x = \pm 1/2 \text{ como}$$

$$x > 0 \Rightarrow x = \frac{1}{2} \Rightarrow A\left(\frac{1}{2}, 2\right)$$

$$B:\ \begin{cases} y = \frac{1}{x} \\ y = \frac{x}{4} \end{cases} \Rightarrow \frac{1}{x} = \frac{x}{4} \Rightarrow 4 = x^2 \Rightarrow x = \pm 2 \Rightarrow x = 2$$

$$\Rightarrow B(2, 1/2).$$

$$A_{Total} = A_1 + A_2 = \int_0^{\frac{1}{2}} \left(4x - \frac{x}{4}\right) dx + \int_{1/2}^{2} \left(\frac{1}{x} - \frac{x}{4}\right) dx = \int_0^{\frac{1}{2}} \frac{15x}{4}\, dx + \int_{\frac{1}{2}}^{2} \left(\frac{1}{x} - \frac{x}{4}\right) dx$$

$$= \left[\frac{15x^2}{8}\right]_0^{1/2} + \left[\ln|x| - \frac{x^2}{8}\right]_{1/2}^{2} = \left(\frac{15}{32} - 0\right) + \left(\ln 2 - \frac{1}{2}\right) - \left(\ln\left|\frac{1}{2}\right| - \frac{1}{32}\right) =$$

$$= \frac{15}{32} + \ln 2 - \frac{1}{2} - \ln\left(\frac{1}{2}\right) + \frac{1}{32} = \frac{16}{32} - \frac{1}{2} + \ln\left(\frac{2}{1/2}\right) = 0 + \ln|4| \Rightarrow$$

$$\Rightarrow \boxed{A_T = \ln 4 \quad u^2}$$