SOLUCIONES

1º

a) $\lim_{x\to 0} (\cos x)^{\frac{1}{\operatorname{sen} x}} = 1^{\infty} \Rightarrow A = \lim_{x\to 0} \frac{1}{\operatorname{sen} x}\cdot(\cos x - 1) =$

$\lim_{x\to 0} \frac{\cos x - 1}{\operatorname{sen} x} = \frac{0}{0} \overset{L'H}{\Rightarrow} e^{A} = e^{\lim_{x\to 0} \frac{-\operatorname{sen} x}{\cos x}} = e^{0} = \underline{1}.$

b) $\lim_{x\to 0}\left(\frac{1}{\tan x} - \frac{1}{x}\right) = \lim_{x\to 0} \frac{x - \tan x}{x\cdot\tan x} = \frac{0}{0} \overset{L'H}{\Rightarrow} \lim_{x\to 0} \frac{1 - \frac{1}{\cos^2 x}}{\tan x + \frac{x}{\cos^2 x}} =$

$= \lim_{x\to 0} \frac{\frac{\cos^2 x - 1}{\cancel{\cos^2 x}}}{\frac{\cos^2 x \tan x + x}{\cancel{\cos^2 x}}} = \lim_{x\to 0} \frac{\cos^2 x - 1}{\cos^2 x\cdot \tan x + x} = \frac{0}{0} \Rightarrow L'H:$

$\lim_{x\to 0} \frac{2\cos x\cdot(-\operatorname{sen} x)}{2\cos x(-\operatorname{sen} x)\tan x + \cos^2 x\cdot\frac{1}{\cos^2 x} + 1} = \frac{0}{2} = \underline{0}$

2º

a) $\int \frac{5\operatorname{sen} x}{1+\cos^2 x}\,dx \Rightarrow t = \cos x \Rightarrow dt = -\operatorname{sen} x\,dx \Rightarrow dx = \frac{-dt}{\operatorname{sen} x} \Rightarrow$

$\Rightarrow \int \frac{5\,\cancel{\operatorname{sen} x}\cdot\frac{-dt}{\cancel{\operatorname{sen} x}}}{1+t^2} = -5\int\frac{dt}{1+t^2} = -5\arctan t = \underline{-5\arctan(\cos x) + C}$

b) $\int (x^2-1)\ln x\,dx$: $u = \ln x$, $dv = (x^2-1)\,dx \Rightarrow du = \frac{1}{x}\,dx$, $v = \frac{x^3}{3} - x$

$\int (x^2-1)\ln x\,dx = \ln x\cdot\left(\frac{x^3}{3} - x\right) - \int\left(\frac{x^3}{3} - x\right)\cdot\frac{1}{x}\,dx = \ln x\cdot\left(\frac{x^3}{3} - x\right) - \int\left(\frac{x^2}{3} - 1\right)dx$

$= \underline{\ln x\cdot\left(\frac{x^3}{3} - x\right) - \left(\frac{x^3}{9} - x\right) + C}$

c) $\int \frac{7\cdot dx}{8+25x^2} = \int \frac{7}{8\left(1+\frac{25}{8}x^2\right)}\,dx = \frac{7}{8}\int\frac{dx}{1+\left(\frac{5}{\sqrt{8}}x\right)^2} = \underline{\frac{7}{8}\cdot\frac{\sqrt{8}}{5}\arctan\left(\frac{5x}{\sqrt{8}}\right) + C}$

d) $I=\int\left(\frac{2\ln x}{x}+\ln x\right)dx=\int\frac{2\ln x}{x}dx+\int\ln x\,dx \Rightarrow$

$\int\frac{2\ln x}{x}dx$: $\ln x=t \Rightarrow \frac{1}{x}dx=dt \Rightarrow dx=x\,dt \Rightarrow$

$\Rightarrow \int\frac{2t}{\cancel{x}}\cdot\cancel{x}\,dt=\int 2t\,dt=t^2=(\ln x)^2$

$\int\ln x\,dx$: $u=\ln x,\ dv=dx \Rightarrow du=\frac{1}{x}dx,\ v=x \Rightarrow$

$\Rightarrow \int\ln x\,dx=\ln x\cdot x-\int x\cdot\frac{1}{x}dx=x\ln x-\int dx=x\ln x-x$

En conclusión $\underline{I=(\ln x)^2+x\cdot\ln x-x+C}$

e) $\int 3\sqrt{2x+1}\,dx$: $2x+1=t^2 \Rightarrow 2\,dx=2t\,dt \Rightarrow dx=t\,dt \Rightarrow$

$\Rightarrow \int 3\cdot t\cdot t\cdot dt=3\int t^2\cdot dt=3\cdot\frac{t^3}{3}=t^3=\underline{\left(\sqrt{2x+1}\right)^3+C}$

3º a) Teoría

b) $f(x)$ y $g(x)$ se cortan en un punto si $f(x)=g(x) \Rightarrow$

$\Rightarrow H(x)=f(x)-g(x)=0$ tiene alguna solución:

$H(x)$ es continua y derivable por ser suma de funciones continuas y derivables. $H(x)=\ln(1+x^2)-1$

$H(0)=-1<0$
$H(2)=\ln 5-1>0$

Aplicando el teorema de Bolzano a la función $H(x)$ en el intervalo $[0,2]$:

$\exists\ c\in(0,2):\ H(c)=0 \Leftrightarrow \exists\ c\in(0,2):\ f(c)=g(c).$

$\Leftrightarrow$ $f(x)$ y $g(x)$ se cortan en $c\in(0,2)$.

4º Dominio $f(x)$: $\mathbb{R}^+$. $f(x) = \dfrac{\ln x}{x}$

$$f'(x) = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2} = 0 \Rightarrow 1-\ln x = 0 \Rightarrow \ln x = 1 \Rightarrow x = e.$$

Signo $f'(x)$:

$f'>0$ $\quad$ $f'<0$

0 $\nearrow$ e $\searrow$ ∞

Crece $(0,e)$; decrece (e,∞) $\quad$ En $x=e$ se alcanza un máximo $(e, \frac{1}{e})$

$$f''(x) = \frac{-\frac{1}{x}\cdot x^2 - (1-\ln x)\cdot 2x}{x^4} = \frac{-x-2x(1-\ln x)}{x^4} = \frac{x\left[-1-2(1-\ln x)\right]}{x^{\cancel{4}}}$$

$$f''(x) = \frac{-1-2+2\ln x}{x^3} = \frac{-3+2\ln x}{x^3} = 0 \Rightarrow -3+2\ln x = 0$$

$$\Rightarrow 2\ln x = 3 \Leftrightarrow \ln x = \frac{3}{2} \Rightarrow x = e^{3/2} = \sqrt{e^3} = e\cdot\sqrt{e}.$$

Concavidad:

$f''<0$ $\quad$ $f''>0$

0 $\cap\,(-)$ $\sqrt{e^3}$ $\cup\,(+)$ ∞

Convexa: $(0, \sqrt{e^3})$ $\quad$ Cóncava $(\sqrt{e^3}, \infty)$ $\quad$ P.I en $x = \sqrt{e^3}$

5º
$$f(x) = \begin{cases} x^2-bx+c & x\le 0 \\ ae^x & x>0 \end{cases}$$

Para $x\neq 0$ es continua y derivable.

Para $x=0$: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) \Leftrightarrow$

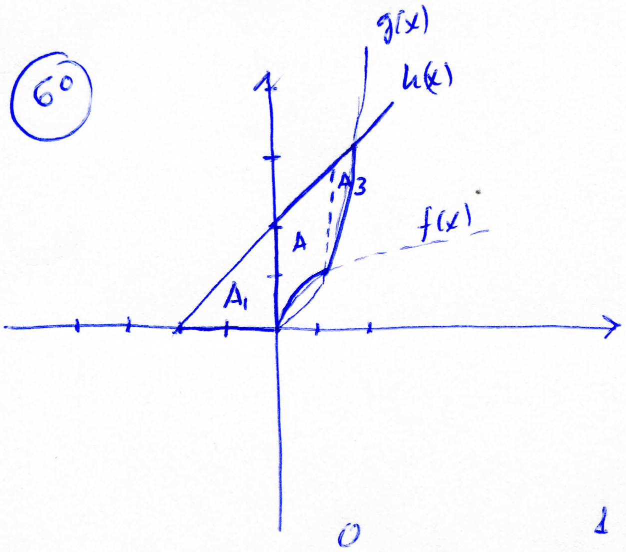
$\Leftrightarrow \boxed{c = a}$ $\quad$ Para este caso: $f'(0)^- = (2x-b)_{x=0} = -b$

$f'(0)^+ = (ae^x)_{x=0} = a \Rightarrow \boxed{-b = a}$

como la recta tangente en $x=-2$ debe tener pendiente 2:

$$f'(-2)=2 \iff f'(-2)=(2x-b)_{x=-2} = \boxed{-4-b=2} \Rightarrow \boxed{b=-6,\ a=6,\ c=6}$$

60



$f(x)=g(x) \Rightarrow x=1$

$g(x)=h(x) \Rightarrow x=2$

$h(x)=$ eje X $\Rightarrow x=-2$

$$\text{Área} = \int_{-2}^{0}(x+2)\,dx + \int_{0}^{1}\left[(x+2)-\sqrt{x}\right]dx + \int_{1}^{2}\left[(x+2)-x^2\right]dx =$$

$$= 2 + \left[\frac{x^2}{2}+2x-\frac{2\sqrt{x^3}}{3}\right]_0^1 + \left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_1^2 =$$

es un triángulo

$$= 2 + \frac{11}{6} + \frac{7}{6} = \frac{12+11+7}{6} = \frac{30}{6} = 5\ u^2$$