ALGEBRA LINEAL

1º) $AXB - C = D \Rightarrow AXB = D + C \Rightarrow X = A^{-1}(D+C)\cdot B^{-1}$

$$A^{-1} = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 1/2 & -1 \end{pmatrix} \qquad D+C = \begin{pmatrix} -1 & 6 & -7 \\ 5 & -2 & 3 \end{pmatrix}$$

$$\Rightarrow X = \begin{pmatrix} 7 & -6 & -8 \\ -5 & 4 & 8 \end{pmatrix}$$

2º) $$\begin{vmatrix} a-x & b-y & c-z \\ 2a & 2b & 2c \\ p & q & r \end{vmatrix} = 2\begin{vmatrix} a-x & b-y & c-z \\ a & b & c \\ p & q & r \end{vmatrix} = 2\left[\begin{vmatrix} a & b & c \\ a & b & c \\ p & q & r \end{vmatrix} + \begin{vmatrix} -x & -y & -z \\ a & b & c \\ p & q & r \end{vmatrix}\right]$$

$$= 2\cdot\left[0 + (-1)\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix}\right] = 2\cdot(-2) = \underline{\underline{-4}}$$

$$\begin{vmatrix} x+y+z & 2y & z \\ a+b+c & 2b & c \\ p+q+r & 2q & r \end{vmatrix} = 2\begin{vmatrix} x+y+z & y & z \\ a+b+c & b & c \\ p+q+r & q & r \end{vmatrix} \underset{C_1 - C_2 - C_3}{=} 2\cdot\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = \underline{\underline{4}}$$

3º) $$A = \begin{pmatrix} m & -1 & 4 \\ 3 & m & 0 \\ -1 & 0 & 1 \end{pmatrix} \Rightarrow |A| = m^2 + 4m + 3 = (m+1)(m+3)$$

Si $m \neq -1, -3 \Rightarrow rg\,A = 3 = rg\,A^* \Rightarrow$ $\underline{\underline{\text{S.C.D.}}}$

Si $m=-1 \Rightarrow rg\, A = rg\begin{pmatrix} 3 & 0 \\ -1 & 1 \end{pmatrix} = 2$ y $rg\, A^* = rg\begin{pmatrix} -1 & -1 & 4 \\ 3 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} = 2$

pues $F_1 = 4F_3 + F_2$ ó $|\ldots| = 0 \Rightarrow$ $\underline{\underline{SCI}}$

Si $m = -3$; $rg\, A = 2$ y $rg\, A^* = rg\begin{pmatrix} -3 & -1 & 4 \\ 3 & -3 & 0 \\ -1 & 0 & 1 \end{pmatrix} = 2$ pues $C_3 = -C_2 - C_1$ ó $|\ldots| = 0$.

$\Rightarrow$ $\underline{\underline{S.C.I}}$.

Para $m = 2 \Rightarrow SCD \Rightarrow$ Regla de Cramer:

$$x = \frac{\begin{vmatrix} -1 & -1 & 4 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix}}{15} \qquad y = \frac{\begin{vmatrix} 2 & -1 & 4 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{vmatrix}}{15} \qquad z = \frac{\begin{vmatrix} 2 & -1 & -1 \\ 3 & 2 & 0 \\ -1 & 0 & 1 \end{vmatrix}}{15}$$

GEOMETRÍA

10) Recta paralela a dos planos α y β que son secantes pues $\vec{n}_\alpha = (1,-3,1)$ y $\vec{n}_\beta(2,-1,3)$ es $r: \begin{cases} x - 3y + z = 0 \\ 2x - y + 3z = 5 \end{cases}$

$d(r,\pi)$: $\vec{d}_r = \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 2 & -1 & 3 \end{vmatrix} = (-8,-1,5)$

Como $r \subset \alpha \Rightarrow d(r,\alpha) = 0$.

2º)

a) $r \equiv \begin{cases} x = 1+2\lambda \\ y = 1+\lambda \\ z = \lambda \end{cases}$ $s \equiv \begin{cases} x = \lambda \\ y = -\lambda \\ z = 3 \end{cases}$ $\vec{d_r} = (2,1,1)$, $\vec{d_s} = (1,-1,0)$ $\Rightarrow \vec{d_r} \not\parallel \vec{d_s}$

$A_r(1\,1\,0)$ $B_s(0\,0\,3)$ $\Rightarrow \overrightarrow{A_rB_s} = (-1,-1,3)$

$\{\overrightarrow{A_rB_s}, \vec{d_r}, \vec{d_s}\} = \begin{vmatrix} -1 & -1 & 3 \\ 2 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} \neq 0 \Rightarrow$ r y s se cruzan.

b) Al cruzarse, no hay un plano que las contenga.

c) $\text{dist}(r,s) = \dfrac{|\{\overrightarrow{A_rB_s}, \vec{d_r}, \vec{d_s}\}|}{\|\vec{d_r} \times \vec{d_s}\|} = \dfrac{11}{\sqrt{11}} = \underline{\underline{\sqrt{11}\ u}}$

$\begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = (1,+1,-3)$

3º)

a) $\overrightarrow{AB} = (0,3,1)$ $\overrightarrow{AC} = (1,2,0)$ $\pi \equiv \begin{vmatrix} x-1 & y+1 & z-1 \\ 0 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow -2(x-1) + 1(y+1) - 3(z-1) = 0 \Rightarrow \pi \equiv \underline{-2x+y-3z+6=0}$

b) Corte con OX: $\begin{cases} \pi \\ y=0 \\ z=0 \end{cases} \Rightarrow x = 3 \rightarrow P(3,0,0)$

$\overrightarrow{PQ}(-3,-6,0)$

Corte con OY: $\begin{cases} \pi \\ x=0 \\ z=0 \end{cases} \Rightarrow y = -6 \rightarrow Q(0,-6,0)$

$\overrightarrow{PR}(-3,0,2)$

Corte con OZ: $\begin{cases} \pi \\ x=0 \\ y=0 \end{cases} \Rightarrow z = 2 \rightarrow R(0,0,2)$

Área PQR: $\text{Área} = \frac{1}{2} \|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \frac{1}{2} \cdot \sqrt{(-12)^2 + 6^2 + (-18)^2} = \frac{1}{2} \cdot \sqrt{504}$

$= \frac{1}{2} \cdot 2\sqrt{126} = \sqrt{126} = \underline{11'22\,u^2}$

$\begin{vmatrix} i & j & k \\ -3 & -6 & 0 \\ -3 & 0 & 2 \end{vmatrix} = (-12, 6, -18)$

ANÁLISIS

(*)

$$1^{o})\ a)\ \int \frac{3x-2}{x^2-3x+2}\,dx = \int \frac{A}{x-1}+\frac{B}{x-2}\,dx = \int\left(\frac{-1}{x-1}+\frac{4}{x-2}\right)dx = (1)$$

$$\frac{3x-2}{x^2-3x+2} = \frac{A}{x-1}+\frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)} \Rightarrow 3x-2 = A(x-2)+B(x-1)$$

Si $x=1 \Rightarrow A=-1$ (*)

Si $x=2 \Rightarrow B=4$

$$(1) = -\ln|x-1| + 4\ln|x-2| + C$$

$$b)\ \int \frac{x^2-2x}{e^x}\,dx = \int (x^2-2x)\,e^{-x}\,dx = (x^2-2x)(-e^{-x}) - \int -e^{-x}\cdot(2x-2)\,dx$$

$u = x^2-2x \rightarrow du = (2x-2)\,dx$

$dv = e^{-x}dx \rightarrow v = -e^{-x}$

$$= -(x^2-2x)e^{-x} + \int e^{-x}(2x-2)\,dx = -(x^2-2x)e^{-x} - \left[(2x-2)(-e^{-x}) - \int e^{-x}\,2dx\right]$$

$u = 2x-2 \rightarrow du = 2\,dx$

$dv = e^{-x}dx \rightarrow v = -e^{-x}$

$$= -(x^2-2x)e^{-x} + (2x-2)e^{-x} + 2\int e^{-x}dx = -(x^2-2x)e^{-x} + (2x-2)e^{-x} - 2e^{-x} =$$

$$= e^{-x}(-x^2+2x+2x-2-2) = e^{-x}\cdot(-x^2+4x-4) = -e^{-x}(x-2)^2 + C$$

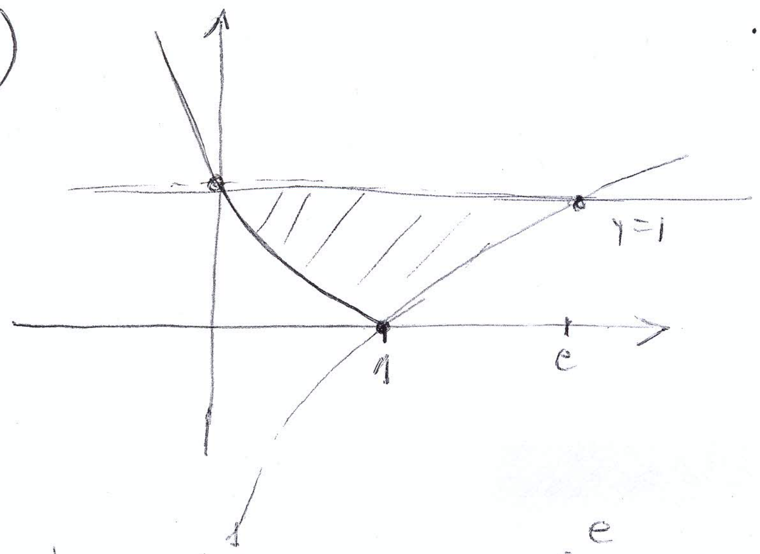
$$c)\ \int \frac{\sqrt{x}}{x+1}\,dx = \int \frac{t}{t^2+1}\cdot 2t\,dt = \int \frac{2t^2}{t^2+1}\,dx = \int\left(2 - \frac{2}{t^2+1}\right)dt$$

$x = t^2 \rightarrow 2t\,dt = dx$

$$\begin{array}{l|l} 2t^2 & t^2+1 \\ \hline -2t^2-2 & 2 \\ \hline -2 & \end{array}$$

$$= 2t - 2\arctan t = 2\sqrt{x} - 2\arctan\sqrt{x} + C$$

2º)



$$\left.\begin{array}{l} y=1 \\ y=\ln x \end{array}\right\} \rightarrow \ln x = 1$$
$$\underline{x=e}$$

$$\text{Área} = \int_0^1 1-(x-1)^2\,dx + \int_1^e 1-\ln x\,dx = \left[x-\frac{(x-1)^3}{3}\right]_0^1 + \left[x\right]_1^e - \int_1^e \ln x\,d$$

$$= \frac{2}{3} + e - 1 - 1 = \boxed{e - \frac{4}{3}\,u^2}$$

$$\int \ln x\,dx = x\ln x - \int dx = x\ln x - x = x(\ln x - 1)$$

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = dx \rightarrow v = x$

3º) $f(x) = x\cdot e^{\frac{1}{x}} \rightarrow f'(x) = 1\cdot e^{\frac{1}{x}} + x\cdot e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right) = e^{\frac{1}{x}}\left(1-\frac{1}{x}\right)$

$f'(x) = 0 \Rightarrow 1 - \frac{1}{x} = 0 \rightarrow x = 1$. Punto crítico

$f'(x)>0$	0	$f'(x)<0$	1	$f'(x)>0$
$\nearrow$		$\searrow$		$\nearrow$

En $(1, f(1))$ hay mínimo

Crece: $(-\infty, 0) \cup (1, \infty)$

Decrece: $(0, 1)$

A.V: $\lim_{x\to 0} f(x) = 0\cdot\infty \Rightarrow \lim_{x\to 0} x\cdot e^{1/x} = \lim_{x\to 0} \frac{e^{1/x}}{1/x} \underset{\frac{\infty}{\infty}}{=} \text{L'H} \lim_{x\to 0} \frac{\left(-\frac{1}{x^2}\right)e^{1/x}}{\left(-1/x^2\right)} = \infty$

$\boxed{x=0}$ A.V.

A.H: $\lim_{x\to\infty} f(x) = \infty\cdot 1 = \infty$

A.O: $m = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} e^{1/x} = 1$ $\quad n = \lim_{x\to\infty}\left(xe^{1/x} - x\right) = \lim_{x\to\infty} x\left(e^{1/x} - 1\right) =$

$$= \infty \cdot 0 \Rightarrow \lim_{x\to\infty} \frac{e^{1/x} - 1}{1/x} \overset{L'H}{=} \lim_{x\to\infty} \frac{\left(-\frac{1}{x^2}\right) e^{1/x}}{\left(-1/x^2\right)} = 1$$

$\Rightarrow$ A. Oblicua: $\boxed{y = x + 1}$