RESOLUCIÓN

1º) $X \cdot A = 2 \cdot X + B^2$ $\qquad A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

$$XA - 2X = B^2 \Rightarrow X \cdot A - X \cdot 2 = B^2$$

$$X \cdot (A - 2I) = B^2 \Rightarrow X \cdot (A-2I) \cdot (A-2I)^{-1} = \underline{B^2 \cdot (A-2I)^{-1}}$$

siempre que existe $(A-2I)^{-1}$:

$$\det(A-2I) = \det\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -3 \end{pmatrix} = -2 \neq 0 \Rightarrow \exists \; (A-2I)^{-1}.$$

$$\left.\begin{matrix} (A-2I)^{-1} = \begin{pmatrix} -\frac{3}{2} & 0 & -\frac{1}{2} \\ 0 & -1 & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \\ B^2 = \begin{pmatrix} 4 & 12 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix} \end{matrix}\right\} \Rightarrow X = \underline{\begin{pmatrix} -6 & -12 & -2 \\ 0 & -16 & 0 \\ -8 & 0 & -8 \end{pmatrix}}$$

2º) $$\operatorname{rg}\begin{pmatrix} k & k-1 & k(k-1) \\ k & 1 & k \\ k & 1 & k-1 \end{pmatrix} \xrightarrow{F_3 - F_1 \,,\, F_2 - F_1} = \operatorname{rg}\begin{pmatrix} k & k-1 & k(k-1) \\ 0 & 2-k & 2k-k^2 \\ 0 & 2-k & -k^2+2k-1 \end{pmatrix} =$$

$$\xrightarrow{F_3 - F_2} = \operatorname{rg}\begin{pmatrix} k & k-1 & k(k-1) \\ 0 & 2-k & 2k-k^2 \\ 0 & 0 & +1 \end{pmatrix} \Rightarrow \begin{cases} \text{Si } k \neq 0,2 \Rightarrow \operatorname{rg} A = 3 \\ \text{Si } k = 0 \Rightarrow \operatorname{rg} A = 2 \\ \text{Si } k = 2 \Rightarrow \operatorname{rg} A = 2 \end{cases}$$

3°)

a)
$$\begin{vmatrix} \frac{z}{2} & z+7 & 3 \\ \frac{y}{2} & y & 3 \\ \frac{x}{2} & x-3 & 3 \end{vmatrix} = \frac{1}{2} \cdot 3 \cdot \begin{vmatrix} z & z+7 & 1 \\ y & y & 1 \\ x & x-3 & 1 \end{vmatrix} =$$

$$= \frac{3}{2} \cdot \left[\underbrace{\begin{vmatrix} z & z & 1 \\ y & y & 1 \\ x & x & 1 \end{vmatrix}}_{=0} + \begin{vmatrix} z & 7 & 1 \\ y & 0 & 1 \\ x & -3 & 1 \end{vmatrix} \right] = \frac{3}{2} \cdot \begin{vmatrix} z & 7 & 1 \\ y & 0 & 1 \\ x & -3 & 1 \end{vmatrix} =$$

$$= \frac{3}{2} \cdot \begin{vmatrix} x & -3 & 1 \\ y & 0 & 1 \\ z & 7 & 1 \end{vmatrix} (-1) = -\frac{3}{2} \cdot 6 = \underline{\underline{-9}}$$

b)
$$\begin{vmatrix} x & -3 & 1 & 2 \\ y & 0 & 1 & 2 \\ z & 7 & 1 & 2 \\ 0 & 6 & 0 & 2 \end{vmatrix} \overset{c_2 + c_4(-3)}{=} \begin{vmatrix} x & -9 & 1 & 2 \\ y & -6 & 1 & 2 \\ z & 1 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 2 \cdot A_{44} + \cancel{0 \cdot A_{43}} + \cancel{0 A_{42}} + \cancel{0 A_{41}}$$

$$= 2 \cdot \begin{vmatrix} x & -9 & 1 \\ y & -6 & 1 \\ z & 1 & 1 \end{vmatrix} = -2 \cdot \left[\begin{vmatrix} x & -3 & 1 \\ y & 0 & 1 \\ z & 7 & 1 \end{vmatrix} + \underbrace{\begin{vmatrix} x & -6 & 1 \\ y & -6 & 1 \\ z & -6 & 1 \end{vmatrix}}_{=0} \right] =$$

$$= 2 \cdot \begin{vmatrix} x & -3 & 1 \\ y & 0 & 1 \\ z & 7 & 1 \end{vmatrix} = 2 \cdot 6 = \underline{\underline{12}}$$

4°) $\det B = \begin{vmatrix} 2 & a-3 \\ b+2 & c \end{vmatrix} = 2c - (a-3)(b+2) = 2c - ab - 2a + 3b + 6$

$\Rightarrow \quad 2c - ab - 2a + 3b + 6 = 8 \Rightarrow \quad 2c - ab - 2a + 3b = 2 \quad (*)$

$A \cdot B = \begin{pmatrix} 6+b & 2a-6+c \\ b+2 & c \end{pmatrix} = BA = \begin{pmatrix} 4 & a-1 \\ 2b+4 & b+2+c \end{pmatrix}$

$$\Rightarrow \left.\begin{array}{r} 6+b = 4 \\ 2a-6+c = a-1 \\ b+2 = 2b+4 \\ c = b+2+c \end{array}\right\} \begin{array}{l} \to \boxed{b=-2} \\ \to a+c=5 \longrightarrow \boxed{a=1} \\ \\ \\ \end{array}$$

De $(*)$: $2c + 2\cancel{a} - 2\cancel{a} - 6 = 2 \to \boxed{c=4}$