1º $\quad rg\begin{pmatrix} 1 & k & -1 & 1 \\ 2 & 1 & -k & 2 \\ 1 & -1 & -1 & k-1 \end{pmatrix} \overset{\substack{F_2+F_1(-2) \\ F_3-F_1}}{=} rg\begin{pmatrix} 1 & k & -1 & 1 \\ 0 & 1-2k & 2-k & 0 \\ 0 & -1-k & 0 & k-2 \end{pmatrix} =$

$\overset{C_2\leftrightarrow C_3}{=} rg\begin{pmatrix} 1 & -1 & k & 1 \\ 0 & 2-k & 1-2k & 0 \\ 0 & 0 & -1-k & k-2 \end{pmatrix} \overset{C_2\leftrightarrow C_3}{=} rg\begin{pmatrix} 1 & -1 & 1 & k \\ 0 & 2-k & 0 & 1-2k \\ 0 & 0 & k-2 & -1-k \end{pmatrix} \Rightarrow$

Si $\quad k \neq 2 \Rightarrow rg\, A = 3$

Si $\quad k = 2 \Rightarrow rg\begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -3 \end{pmatrix} = 2$

2º $\quad rg\, A = rg\begin{pmatrix} 1 & 1 & 1 \\ 0 & a & a \\ a & a & 1 \end{pmatrix} \overset{\substack{C_3-C_2 \\ C_2-C_1}}{=} rg\begin{pmatrix} 1 & 0 & 0 \\ 0 & a & a \\ a & 0 & 1-a \end{pmatrix} \overset{C_3-C_2}{=} rg\begin{pmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ a & 0 & 1-a \end{pmatrix}$

$\Rightarrow$ Si $\quad a \neq 0, 1 \Rightarrow rg\, A = 3$ y $rg\, A^* = 3 \Rightarrow \boxed{SCD}$

Si $\quad a = 0 \Rightarrow rg\, A = rg\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 2$ y $rg\, A^* = rg\begin{pmatrix} 1 & \cancel{0} & 0 & 1 \\ 0 & \cancel{0} & 0 & 2 \\ 0 & \cancel{0} & 1 & 1 \end{pmatrix} = rg\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix} = 3$

pues $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -2 \neq 0 \Rightarrow \boxed{S.I}$

Si $\quad a = 1 \Rightarrow rg\, A = rg\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = rg\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = rg\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = 2.$

$rg\, A^* = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix} = rg\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} = rg\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = 2 \Rightarrow \boxed{SCI}$

pues $C_3 = 2C_2 - C_1$

Para $a=2 \Rightarrow$ SCD $\Rightarrow$ Cramer: $|A| = -2$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 1 \end{vmatrix}}{-2} = \frac{0}{-2} = 0 \qquad y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 2 & 1 & 1 \end{vmatrix}}{-2} = \frac{0}{-2} \qquad z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}}{-2} = \frac{-2}{-2} = 1$$

3º $A \cdot X \cdot B + C = 2D \Rightarrow A \cdot X \cdot B = 2D - C \Rightarrow$

$$\Rightarrow A^{-1} \cdot A \cdot X \cdot B \cdot B^{-1} = A^{-1} \cdot (2D - C) \cdot B^{-1} \Rightarrow$$

$$\Rightarrow X = A^{-1} \cdot (2D - C) \cdot B^{-1} = \begin{pmatrix} \frac{13}{2} & -21 & \frac{9}{2} \\ -\frac{17}{2} & 29 & -\frac{13}{2} \\ -4 & 13 & -3 \end{pmatrix}$$

4º a) $$\begin{vmatrix} 2b & c+3a & a/5 \\ 2e & f+3d & d/5 \\ 2h & i+3g & g/5 \end{vmatrix} = \frac{2}{5} \cdot \begin{vmatrix} b & c+3a & a \\ e & f+3d & d \\ h & i+3g & g \end{vmatrix} =$$

$$= \frac{2}{5} \cdot \left[\begin{vmatrix} b & c & a \\ e & f & d \\ h & i & g \end{vmatrix} + \overset{0}{\overset{\parallel}{\begin{vmatrix} b & 3a & a \\ e & 3d & d \\ h & 3g & g \end{vmatrix}}} \right] = \frac{2}{5} (-1) \cdot \begin{vmatrix} b & a & c \\ e & d & f \\ h & g & i \end{vmatrix} =$$

$$= \frac{2}{5} \cdot (-1) \cdot (-1) \cdot \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \frac{2}{5} \cdot 6 = \boxed{\frac{12}{5}}$$

b) $$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 2 & 2 & 2 \end{vmatrix} \overset{F_1+F_2}{=} \begin{vmatrix} x+y+z & x+y+z & z+x+y \\ z & x & y \\ 2 & 2 & 2 \end{vmatrix} =$$

$$= 2 \cdot (x+y+z) \cdot \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 2 \cdot (x+y+z) \cdot 0 = 0$$

5°
$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \end{vmatrix} \overset{C_4-C_1,\ C_3-C_1,\ C_2-C_1}{=} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1+a & -a & -a & -a \\ 1 & b & 0 & 0 \\ 1 & 0 & c & 0 \end{vmatrix} =$$

$$= 1 \cdot \begin{vmatrix} -a & -a & -a \\ b & 0 & 0 \\ 0 & c & 0 \end{vmatrix} = -a \cdot \begin{vmatrix} 1 & 1 & 1 \\ b & 0 & 0 \\ 0 & c & 0 \end{vmatrix} = -a(c)(-1) \cdot b =$$

$$= \boxed{-abc}$$