SOLUCIONES

1º a) $\begin{vmatrix} 1 & 1 & 1 \\ \frac{3}{2} & 0 & 1 \\ 2x & 2y & 2z \end{vmatrix} = \frac{1}{2} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 3 & 0 & 2 \\ 2x & 2y & 2z \end{vmatrix} = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 3 & 0 & 2 \\ x & y & z \end{vmatrix} = - \begin{vmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix} = \boxed{-5}$

b) $\begin{vmatrix} x-1 & z-1 & y-1 \\ 4 & 3 & 1 \\ 2 & 2 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} x-1 & z-1 & y-1 \\ 4 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2 \cdot \left[\begin{vmatrix} x & z & y \\ 4 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \underbrace{\begin{vmatrix} -1 & -1 & -1 \\ 4 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix}}_{=0} \right] =$

$= 2 \cdot \begin{vmatrix} x & z & y \\ 4 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -2 \cdot \begin{vmatrix} x & y & z \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = -2 \cdot \left[\begin{vmatrix} x & y & z \\ 3+1 & 0+1 & 2+1 \\ 1 & 1 & 1 \end{vmatrix} \right] =$

$= -2 \cdot \left[\begin{vmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix} + \underbrace{\begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}_{=0} \right] = -2 \cdot 5 = \boxed{-10}$

2º a) $|A| = \begin{vmatrix} 1 & m & 1 \\ 1 & 1 & m \\ m & 1 & 1 \end{vmatrix} = m^3 + 2 - 3m = (m-1)^2 (m+2)$

* Si $m \neq 1, -2 \Rightarrow rg\, A = 3 = rg\, A^* \Rightarrow \boxed{SCD}$
* Si $m = 1 \Rightarrow rg\, A = rg \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = 1$ y $rg\, A^* = rg \begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & -4 \\ 1 & 1 & 1 & 1 \end{pmatrix} = 2 \Rightarrow \boxed{S.I}$
* Si $m = -2 \Rightarrow rg\, A = rg \begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & -2 \\ -2 & 1 & 1 \end{pmatrix} = rg \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ -2 & 1 \end{pmatrix} = 2$ y

$C_3 = -C_2 - C_1$

$rg\, A^* = rg \begin{pmatrix} 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 2 \\ -2 & 1 & 1 & -2 \end{pmatrix} = rg \begin{pmatrix} 1 & -2 & 0 \\ 1 & 1 & 2 \\ -2 & 1 & -2 \end{pmatrix} : \begin{vmatrix} 1 & -2 & 0 \\ 1 & 1 & 2 \\ -2 & 1 & -2 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow rg\, A^* = 2 \Rightarrow \boxed{SCI}$

b) * Si $m=0 \Rightarrow SCD \Rightarrow$ Sistema de Cramer.

$$\left.\begin{aligned} x+z&=2 \\ x+y&=-2 \\ y+z&=0 \end{aligned}\right\} \qquad |A|=2 \Rightarrow x=\frac{\begin{vmatrix} 2 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}}{2}=0$$

$$y=\frac{\begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & 1 \end{vmatrix}}{2}=-2 \qquad z=\frac{\begin{vmatrix} 1 & 0 & 2 \\ 1 & 1 & -2 \\ 0 & 1 & 0 \end{vmatrix}}{2}=2$$

* Si $m=-2 \Rightarrow SCI \Rightarrow rg\, A = rg\left(\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & -2 \\ \hline -2 & 1 & 1 \end{array}\right)=2 \Rightarrow$

$$\Rightarrow \boxed{z=\lambda} \Rightarrow \left.\begin{aligned} x-2y+\lambda&=0 \\ x+y-2\lambda&=2 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x-2y&=-\lambda \\ x+y&=2+2\lambda \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow E_2-E_1: \quad 3y=2+3\lambda \Rightarrow y=\frac{2+3\lambda}{3} \Rightarrow \boxed{y=\frac{2}{3}+\lambda}$$

$$\Rightarrow x=2+2\lambda-\left(\frac{2}{3}+\lambda\right)=\boxed{\frac{4}{3}+\lambda}$$

30 a) $A^2=\begin{pmatrix} 1 & 2/7 & 2/7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad A^3=\begin{pmatrix} 1 & 3/7 & 3/7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad A^4=\begin{pmatrix} 1 & 4/7 & 4/7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\Rightarrow A^n=\begin{pmatrix} 1 & n/7 & n/7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow A^{105}=\begin{pmatrix} 1 & 15 & 15 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b) $B^2=\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad B^3=\begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \quad B^4=\begin{pmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow$

$$\Rightarrow B^n=\begin{pmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

4º) i) a) $AXB + C = I \Rightarrow AXB = I - C \Rightarrow A^{-1}AXBB^{-1} = A^{-1}(I-C)B^{-1}$

$$\Rightarrow \boxed{X = A^{-1}\cdot(I-C)\cdot B^{-1}}$$

b) $AX + 2B = 3X + C \Rightarrow AX - 3X = C - 2B \Rightarrow$

$\Rightarrow (A-3I)X = C-2B \Rightarrow (A-3I)^{-1}(A-3I)\cdot X = (A-3I)^{-1}(C-2B) \Rightarrow$

$$\boxed{X = (A-3I)^{-1}\cdot(C-2B)}$$

ii) En a) $X = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$

$$X = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 \end{pmatrix}$$

En b) $X = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow$

$$X = \begin{pmatrix} -3 & -1 \\ -1/2 & 1 \end{pmatrix}$$

5º) x = nº viajeros que pagan el total.
y = " " " 20%
z = " " " 50%

20% de 1500 ⇒ 300
50% de 1500 ⇒ 750

$$\left.\begin{array}{l} x+y+z = 500 \\ 1500x + 300y + 750z = 352.500 \\ y = 2x \end{array}\right\} \xRightarrow{\text{sustitución}} \begin{array}{l} x + 2x + z = 500 \\ 1500x + 600x + 750z = 352.500 \end{array}$$

Eliminamos "x":

$$\Rightarrow \begin{cases} 3x + z = 500 & |\times(-700) \\ 2100x + 750z = 352.500 \end{cases} \qquad 50z = 2500 \Rightarrow \boxed{z = 50}$$

$$3x = 500 - 50 \Rightarrow \boxed{x = 150} \Rightarrow \boxed{y = 300}$$

$\boxed{\text{Extra:}}$ $F_5 - F_4$:

$$\begin{vmatrix} a & a & a & a & a \\ a & b & b & b & b \\ a & b & c & c & c \\ a & b & c & d & d \\ 0 & 0 & 0 & 0 & e-d \end{vmatrix} \quad F_4 - F_3 \Rightarrow$$

$$\Rightarrow \begin{vmatrix} a & a & a & a & a \\ a & b & b & b & b \\ a & b & c & c & c \\ 0 & 0 & 0 & d-c & d-c \\ 0 & 0 & 0 & 0 & e-d \end{vmatrix} \overset{F_3 - F_2}{=} \begin{vmatrix} a & a & a & a & a \\ a & b & b & b & b \\ 0 & 0 & c-b & c-b & c-b \\ 0 & 0 & 0 & d-c & d-c \\ 0 & 0 & 0 & 0 & e-d \end{vmatrix} \Rightarrow$$

$F_2 - F_1$

$$\Rightarrow \begin{vmatrix} a & a & a & a & a \\ 0 & b-a & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b & c-b \\ 0 & 0 & 0 & d-c & d-c \\ 0 & 0 & 0 & 0 & e-d \end{vmatrix} \Rightarrow$$

Matriz triangular:

$$\Rightarrow \underline{a \cdot (b-a) \cdot (c-b) \cdot (d-c) \cdot (e-d)}$$