

$$1^{\circ} \int \frac{2x^2+2x-1}{x^3-x^2} dx = \int \frac{2x^2+2x-1}{x^2 \cdot (x-1)} dx = \int \left( \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1} \right) dx =$$

$$= \int \frac{1}{x^2} dx - \int \frac{1}{x} dx + \int \frac{3}{x-1} dx =$$

$$= \int x^{-2} dx - \ln|x| + 3 \ln|x-1| =$$

$$= \left\{ -\frac{1}{x} - \ln|x| + 3 \ln|x-1| + C \right\}$$

Calculo de A, B, C:

$$2x^2+2x-1 = A(x-1) + Bx(x-1) + Cx^2$$

$$x=0 \rightarrow -A = -1 \Rightarrow A = 1$$

$$x=1 \rightarrow 3 = C$$

$$x=-1 \rightarrow -1 = -2 + 2B + 3 \rightarrow B = -1$$

$$2^{\circ} \int x^2 (\ln x)^2 dx = (\ln x)^2 \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{2 \ln x}{x} dx = \frac{(\ln x)^2 \cdot x^3}{3} - \int \frac{2 \ln x \cdot x^2}{3} dx$$

$$\left. \begin{array}{l} u = (\ln x)^2 \rightarrow du = \frac{2 \ln x}{x} dx \\ v = x^2 dx \Rightarrow v = \frac{x^3}{3} \end{array} \right\} \parallel = \frac{(\ln x)^2 \cdot x^3}{3} - \int \frac{2x^2}{3} \cdot \ln x \cdot dx =$$

$$\left. \begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ v = \frac{2x^2}{3} dx \rightarrow v = \frac{2}{3} \cdot \frac{x^3}{3} = \frac{2x^3}{9} \end{array} \right\} \parallel = \frac{(\ln x)^2 \cdot x^3}{3} - \left[ \frac{2x^3 \ln x}{9} - \int \frac{2x^3}{9} \cdot \frac{1}{x} dx \right] =$$

$$\frac{x^3 (\ln x)^2}{3} - \frac{2x^3 \ln x}{9} + \frac{2}{9} \int x^2 dx = \left\{ \frac{2x^3 \ln x}{3} - \frac{2x^3 \ln x}{9} + \frac{2x^3}{27} + C \right\}$$

$$3^{\circ} \int \frac{\sin^3 x}{\cos^2 x} dx = \int \frac{\sin^3 x}{t^2} \cdot \frac{-dt}{\sin x} = \int \frac{-\sin^2 x}{t^2} \cdot dt = - \int \frac{1-t^2}{t^2} dt$$

$$\cos x = t \rightarrow -\sin x dx = dt$$

$$dx = \frac{-dt}{\sin x} \quad \sin^2 x = 1 - \cos^2 x = 1 - t^2$$

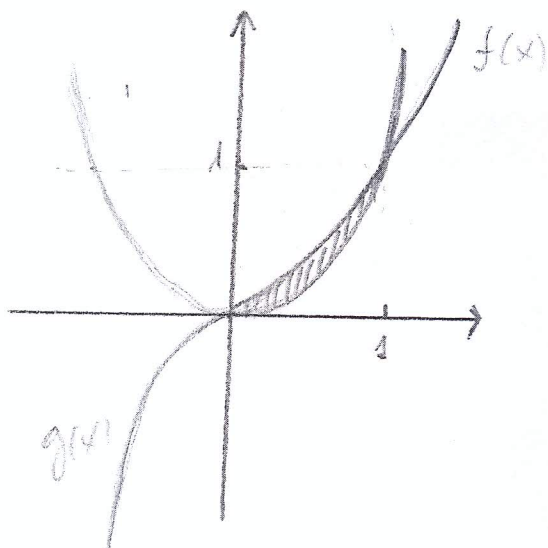
$$\parallel = - \int \left( \frac{1}{t^2} - 1 \right) dt = - \int (t^{-2} - 1) dt =$$

$$= - \left[ \frac{t^{-1}}{-1} - t \right] = t^{-1} + t = \frac{1}{t} + t =$$

$$= \left\{ \frac{1}{\cos x} + \cos x + C \right\}$$

$$(4^{\circ}) \int \frac{(x-1)^2}{\sqrt{x}} dx = \int \frac{x^2 - 2x + 1}{\sqrt{x}} dx = \int \left( \frac{x^2}{\sqrt{x}} - \frac{2x}{\sqrt{x}} + \frac{1}{\sqrt{x}} \right) dx =$$

$$= \int (x^{3/2} - 2x^{1/2} + x^{-1/2}) dx = \boxed{\frac{2}{5} x^{5/2} - \frac{4}{3} x^{3/2} + 2x^{1/2} + C}$$



(5°)

$$f(x) = g(x) \rightarrow x=0, x=1$$

$$\text{Area} = \int_0^1 (x^2 - x^3) dx = \left[ \frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 =$$

$$= \left( \frac{1}{3} - \frac{1}{4} \right) - (0) = \boxed{\frac{1}{12} u^2}$$

$$(6^{\circ}) f(x) = g(x) \Rightarrow x^3 - x^2 + 2 = -x^2 + x + 2 \Rightarrow x^3 - x = 0 \Rightarrow x = 0, 1, -1$$

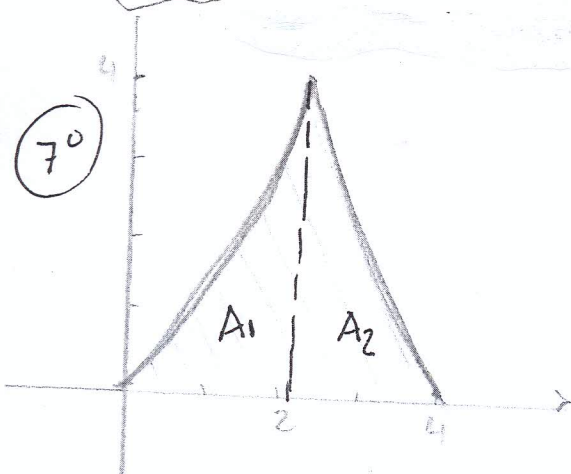
$$A = \int_{-1}^0 (f(x) - g(x)) dx + \int_0^1 (g(x) - f(x)) dx$$

$$f\left(\frac{1}{2}\right) = -\frac{1}{3}$$

$$g\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$= \int_{-1}^0 (x^3 - x) dx + \int_0^1 (x - x^3) dx = \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 =$$

$$= \boxed{\frac{1}{2} u^2}$$



(7°)

$$f(x) = g(x) \rightarrow x=2 \rightarrow A = A_1 + A_2$$

$$A_1 = \int_0^2 x^2 dx = \left[ \frac{x^3}{3} \right]_0^2 = \frac{8}{3} u^2$$

$$A_2 = \int_2^4 (x^2 - 8x + 16) dx = \left[ \frac{x^3}{3} - 4x^2 + 16x \right]_2^4 = \frac{8}{3} u^2$$

$$\boxed{A_T = \frac{16}{3} u^2}$$