

$$(10) \int (x^2+3) e^{3x} dx = (x^2+3) \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} \cdot 2x dx = \frac{(x^2+3)e^{3x}}{3} - \frac{2}{3} \int x e^{3x} dx$$

$$\left. \begin{array}{l} u = x^2+3 \rightarrow du = 2x dx \\ dv = e^{3x} dx \rightarrow v = \int e^{3x} dx = \frac{e^{3x}}{3} \end{array} \right\} \parallel = \frac{(x^2+3)e^{3x}}{3} - \frac{2}{3} \left[\frac{x e^{3x}}{3} - \int \frac{e^{3x}}{3} dx \right]$$

$$\left. \begin{array}{l} u = x \rightarrow du = dx \\ dv = e^{3x} dx \rightarrow v = \frac{e^{3x}}{3} \end{array} \right\} \parallel = \frac{(x^2+3)e^{3x}}{3} - \frac{2x e^{3x}}{9} - \frac{2}{9} \cdot \frac{e^{3x}}{3} + C$$

$$(20) \int \frac{x^4+x+4}{x^3-4x} dx = \int \left(x + \frac{4x^2+x+4}{x^3-4x} \right) dx = \frac{x^2}{2} + \int \frac{4x^2+x+4}{x^3-4x} dx$$

$$\frac{x^4+x+4}{x^3-4x} \parallel \text{Resolvemos } \int \frac{4x^2+x+4}{x^3-4x} dx = \int \frac{4x^2+x+4}{x(x+2)(x-2)} dx$$

$$= \int \left(\frac{A}{x} + \frac{B}{(x+2)} + \frac{C}{(x-2)} \right) dx = \int \left(-\frac{1}{x} + \frac{9/4}{(x+2)} + \frac{11/4}{(x-2)} \right) dx =$$

$$\text{Obtenemos } A, B, C: 4x^2+x+4 = A(x^2-4) + Bx(x-2) + Cx(x+2)$$

$$\text{Si } x=0 \Rightarrow 4 = -4A \Rightarrow A = -1$$

$$\text{Si } x=2 \Rightarrow 22 = 8C \Rightarrow C = 11/4$$

$$\text{Si } x=-2 \Rightarrow 18 = 8B \Rightarrow B = 9/4$$

$$= -\ln x + \frac{9}{4} \ln|x+2| + \frac{11}{4} \ln|x-2| \Rightarrow$$

$$\int \frac{x^4+x+4}{x^3-4x} dx = \frac{x^2}{2} - \ln|x| + \frac{9}{4} \ln|x+2| + \frac{11}{4} \ln|x-2| + C$$

$$(3^o) \int \frac{\cos x}{1 + \sin^2 x} dx = \int \frac{dt}{1+t^2} = \arctan |t| = \underline{\arctan(\sin x) + C}$$

$$\sin x = t \rightarrow \cos x \cdot dx = dt$$

$$(4^o) \int \frac{-x+3}{4x^2+9} dx = \int \frac{-x}{4x^2+9} dx + \int \frac{3}{4x^2+9} dx = (1)$$

Calculamos a) $\int \frac{-x}{4x^2+9} dx = -\frac{1}{8} \int \frac{8x}{4x^2+9} dx = -\frac{1}{8} \ln |4x^2+9|$

b) $\int \frac{3}{4x^2+9} dx = \int \frac{3/9}{\frac{4x^2}{9}+1} dx = \frac{3}{9} \int \frac{1}{\left(\frac{2x}{3}\right)^2+1} dx = \frac{3}{9} \int \frac{\frac{3}{2} dt}{t^2+1} =$

$$t = \frac{2x}{3} \rightarrow dt = \frac{2dx}{3}$$

$$= \frac{3}{9} \cdot \frac{3}{2} \cdot \int \frac{dt}{t^2+1} = \frac{1}{2} \arctan |t| = \frac{1}{2} \arctan \left(\frac{2x}{3} \right)$$

Entonces: $\int \frac{-x+3}{4x^2+9} dx = -\frac{1}{8} \ln |4x^2+9| + \frac{1}{2} \arctan \left(\frac{2x}{3} \right) + C$

$$(5^o) G(x) = \int x \cdot \sqrt{x^2+1} dx = \int x \cdot \sqrt{t} \cdot \frac{dt}{2x} = \frac{1}{2} \int \sqrt{t} dt =$$

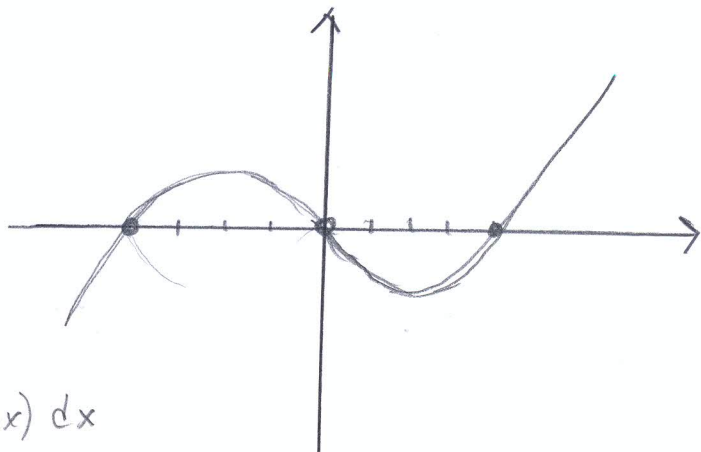
$$x^2+1=t \Rightarrow 2x dx = dt \quad \parallel \quad = \frac{1}{2} \int t^{1/2} dt = \frac{1}{2} \cdot \frac{t^{3/2}}{3/2} = \frac{1}{3} t^{3/2} = \frac{1}{3} \sqrt{t^3} =$$

$$= \frac{1}{3} \sqrt{(x^2+1)^3} = \frac{1}{3} \cdot (x^2+1) \cdot \sqrt{x^2+1} + C = \frac{1}{3} (x^2+1) \cdot \sqrt{x^2+1} + \frac{2}{3}$$

Como $G(0) = 1 \Rightarrow 1 = \frac{1}{3} + C \Rightarrow C = \frac{2}{3}$

6° $f(x) = x^3 - 16x \Rightarrow$

$f(x) = 0 \Rightarrow x = 0, 4, -4$

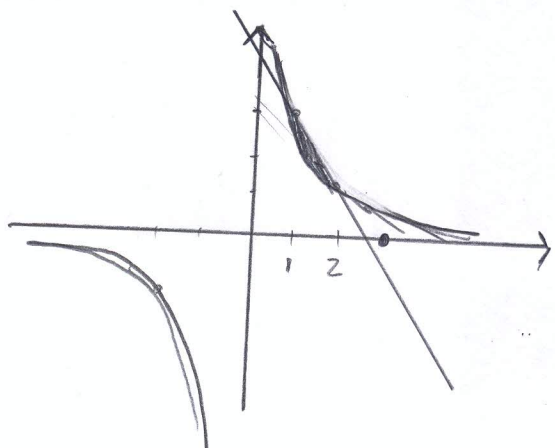


$$\text{Area} = \int_{-4}^0 (x^3 - 16x) dx - \int_0^4 (x^3 - 16x) dx$$

$$= \left[\frac{x^4}{4} - 8x^2 \right]_{-4}^0 - \left[\frac{x^4}{4} - 8x^2 \right]_0^4 = [0 - (-64 - 128)] - [64 - 128] =$$

$$= 64 - (-64) = \boxed{128 \text{ u}^2}$$

7°



$f(x) = g(x) \Rightarrow \frac{2}{x} = 3 - x \Rightarrow$

$$2 = 3x - x^2 \rightarrow x^2 - 3x + 2 = 0$$

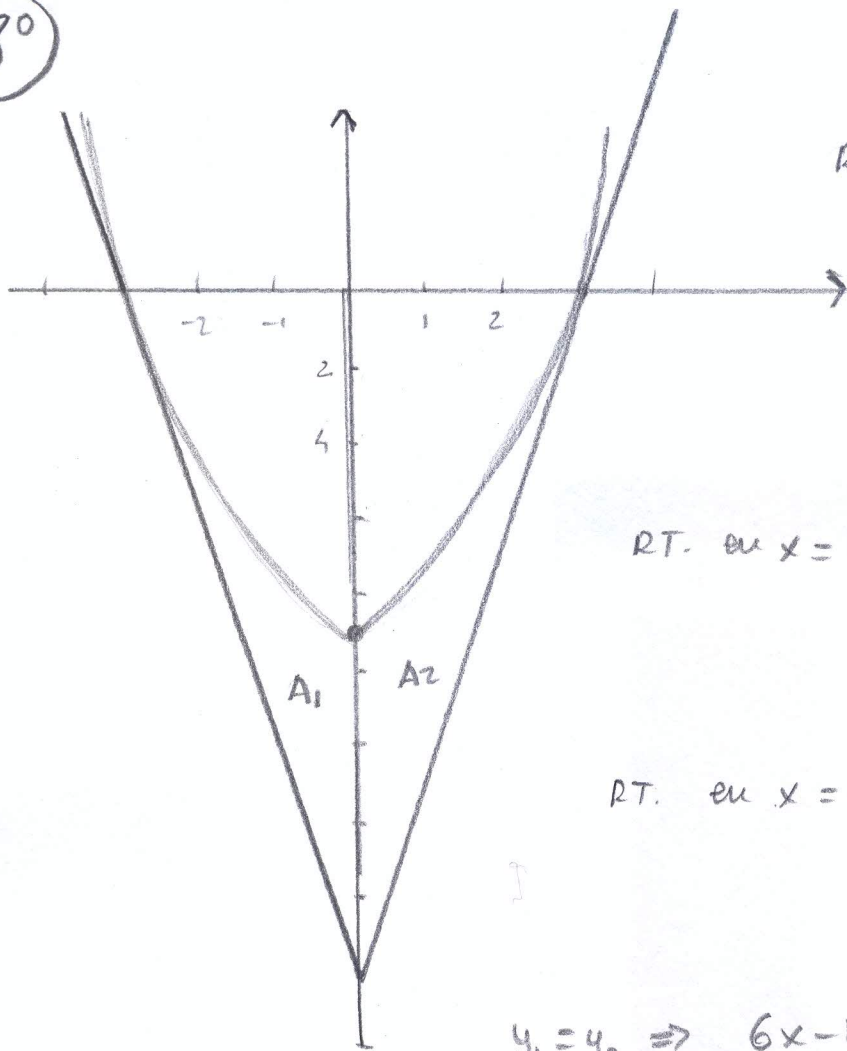
$$x = 1, 2$$

$$\int_1^2 \left((3-x) - \frac{2}{x} \right) dx = \int_1^2 \left(3 - x - \frac{2}{x} \right) dx = \left[3x - \frac{x^2}{2} - 2 \ln|x| \right]_1^2 =$$

$$= (6 - 2 - 2 \ln 2) - \left(3 - \frac{1}{2} - 2 \ln 1 \right) = 4 - 2 \ln 2 - 3 + \frac{1}{2} =$$

$$= \frac{3}{2} - 2 \ln 2 = \boxed{\frac{3}{2} - \ln 4} \approx \underline{\underline{0.1137 \text{ u}^2}}$$

8°



RT. $y - f(a) = f'(a)(x - a)$

$$f(x) = x^2 - 9$$

$$f'(x) = 2x$$

RT. en $x = 3$: $y - 0 = 6(x - 3)$

$$y_1 = 6x - 18$$

RT. en $x = -3$: $y - 0 = -6(x + 3)$

$$y_2 = -6x - 18$$

$$y_1 = y_2 \Rightarrow 6x - 18 = -6x - 18 \rightarrow x = 0$$

$$f(x) = y_1 \rightarrow x = 3$$

$$f(x) = y_2 \rightarrow x = -3$$

$$\begin{aligned} A_T &= A_1 + A_2 = \int_{-3}^0 ((x^2 - 9) - (-6x - 18)) dx + \int_0^3 ((x^2 - 9) - (6x - 18)) dx \\ &= \int_{-3}^0 (x^2 + 6x + 9) dx + \int_0^3 (x^2 - 6x + 9) dx = \left[\frac{x^3}{3} + 3x^2 + 9x \right]_{-3}^0 + \left[\frac{x^3}{3} - 3x^2 + 9x \right]_0^3 \\ &= [0 - (-9 + 27 - 27)] + [(9 - 27 + 27) - 0] = 9 + 9 = 18 \text{ u}^2 \end{aligned}$$